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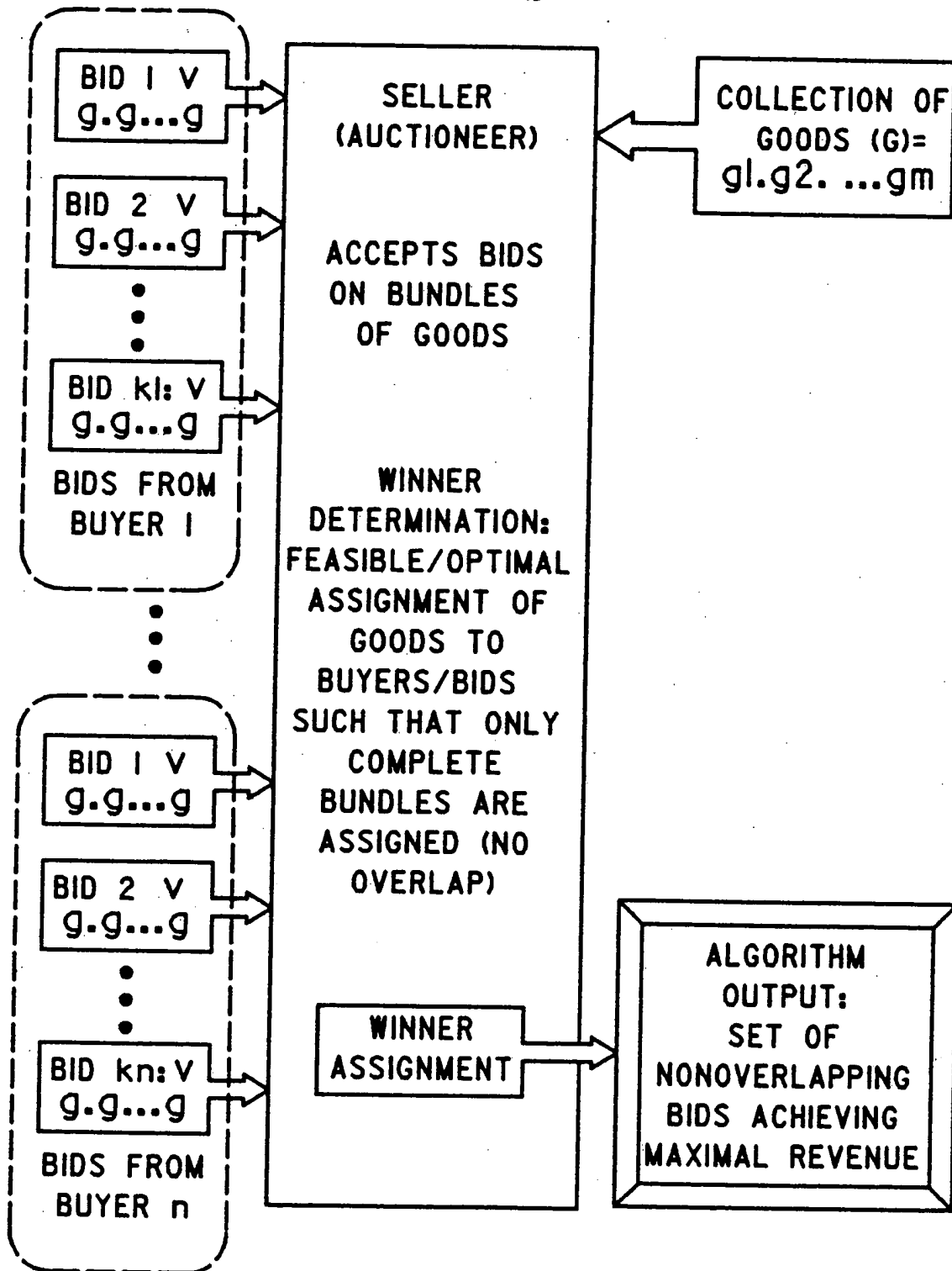


FIG. 1  
(PRIOR ART)

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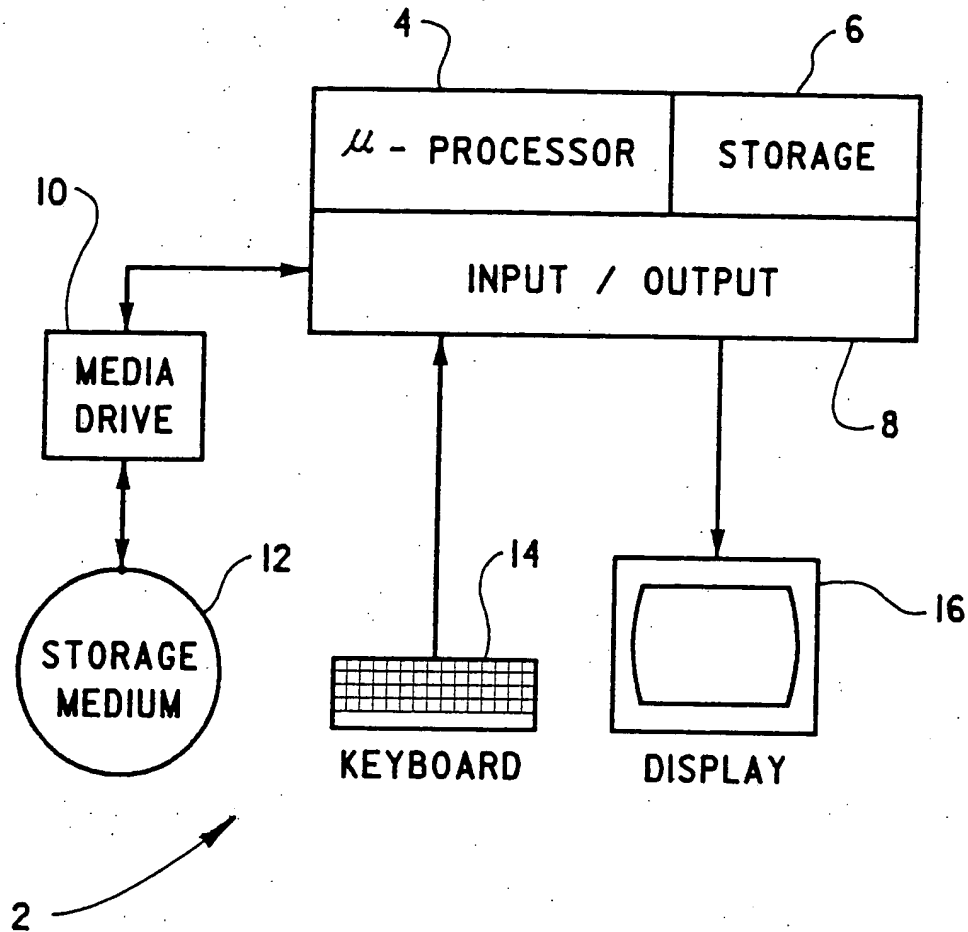
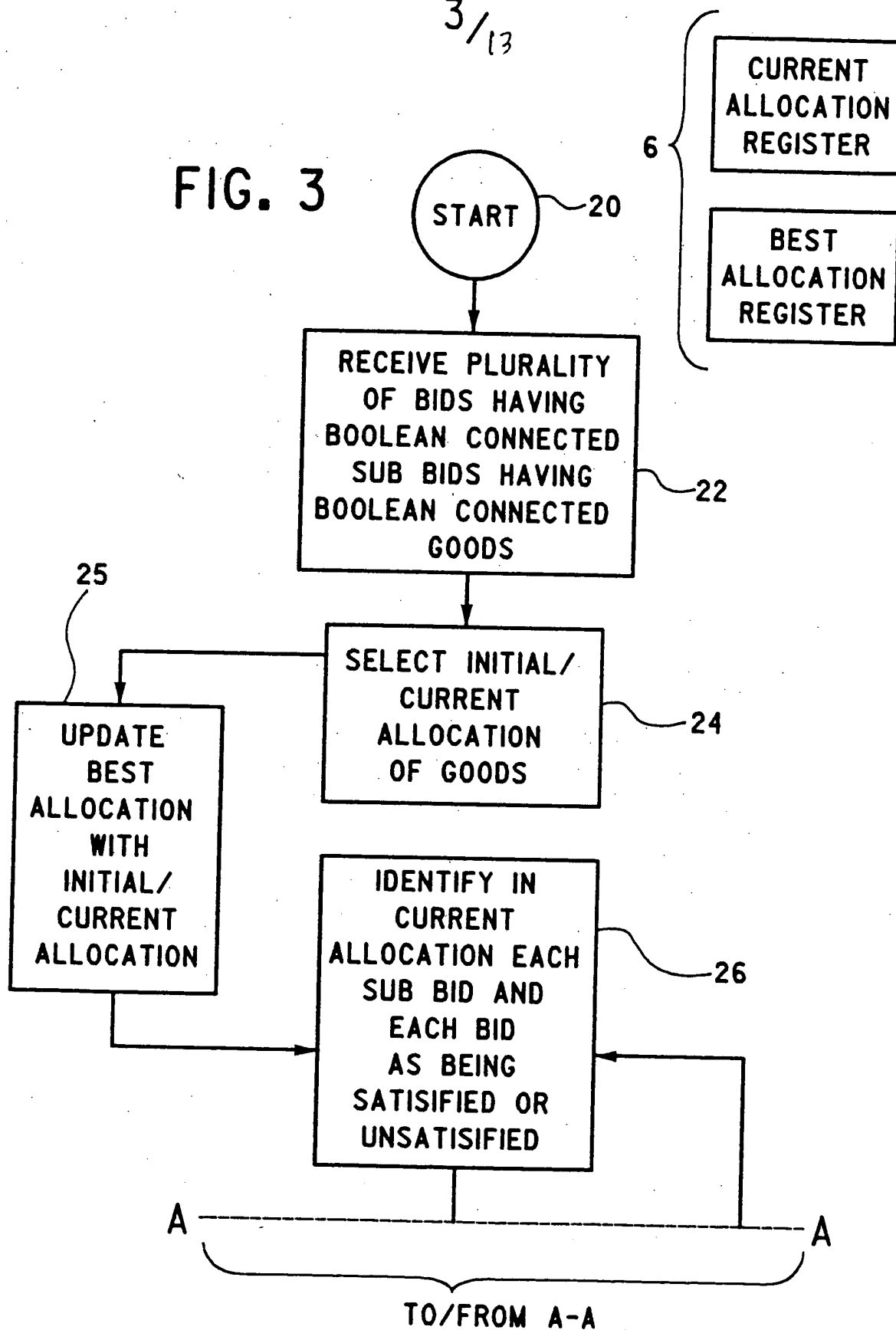
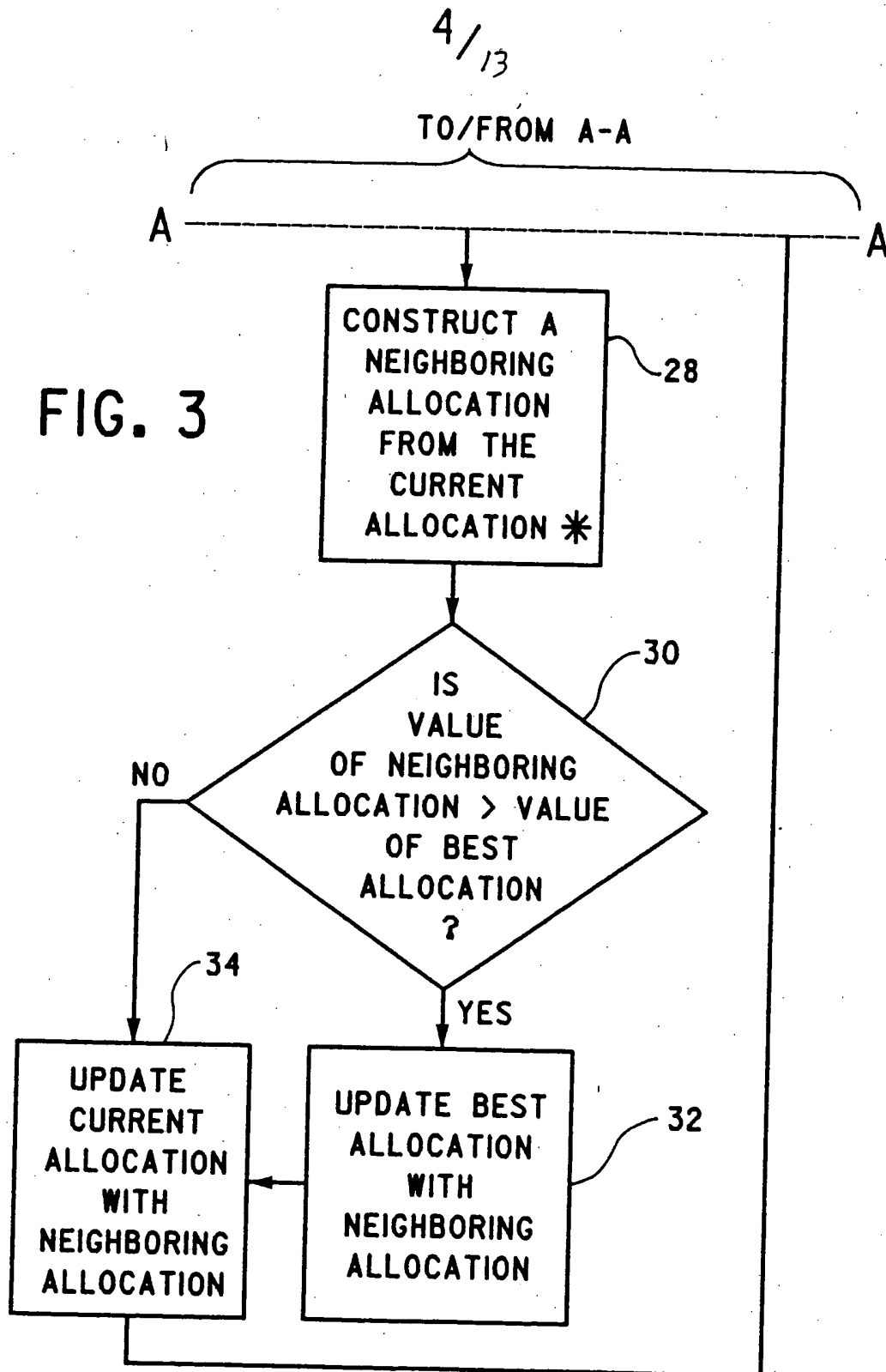


FIG. 2

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FIG. 3





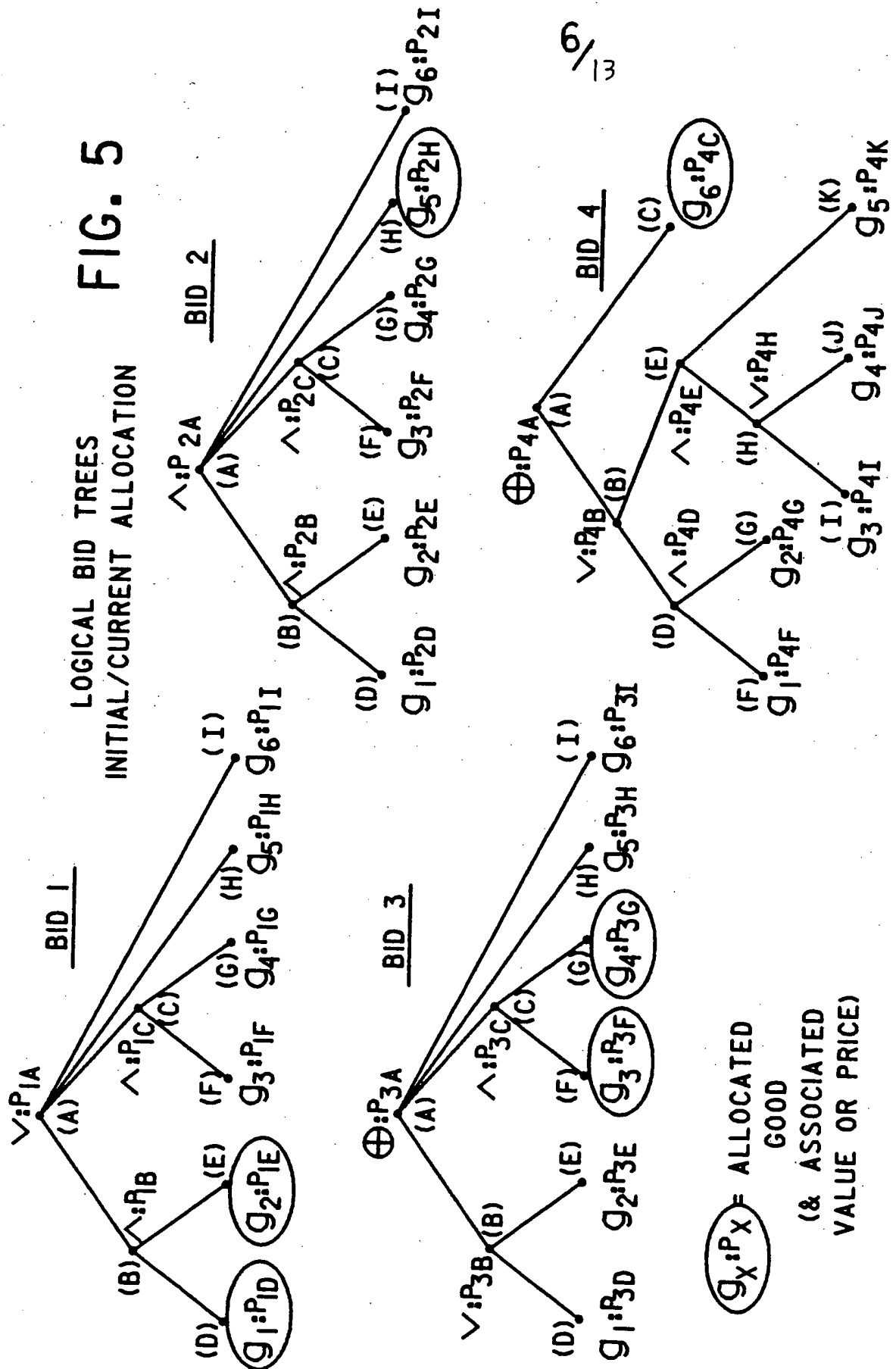
\* REALLOCATE AT LEAST ONE GOOD FROM AT LEAST ONE OF THE SUB BIDS OF AT LEAST ONE BID TO ONE OF THE SUB BIDS OF ANOTHER BID.

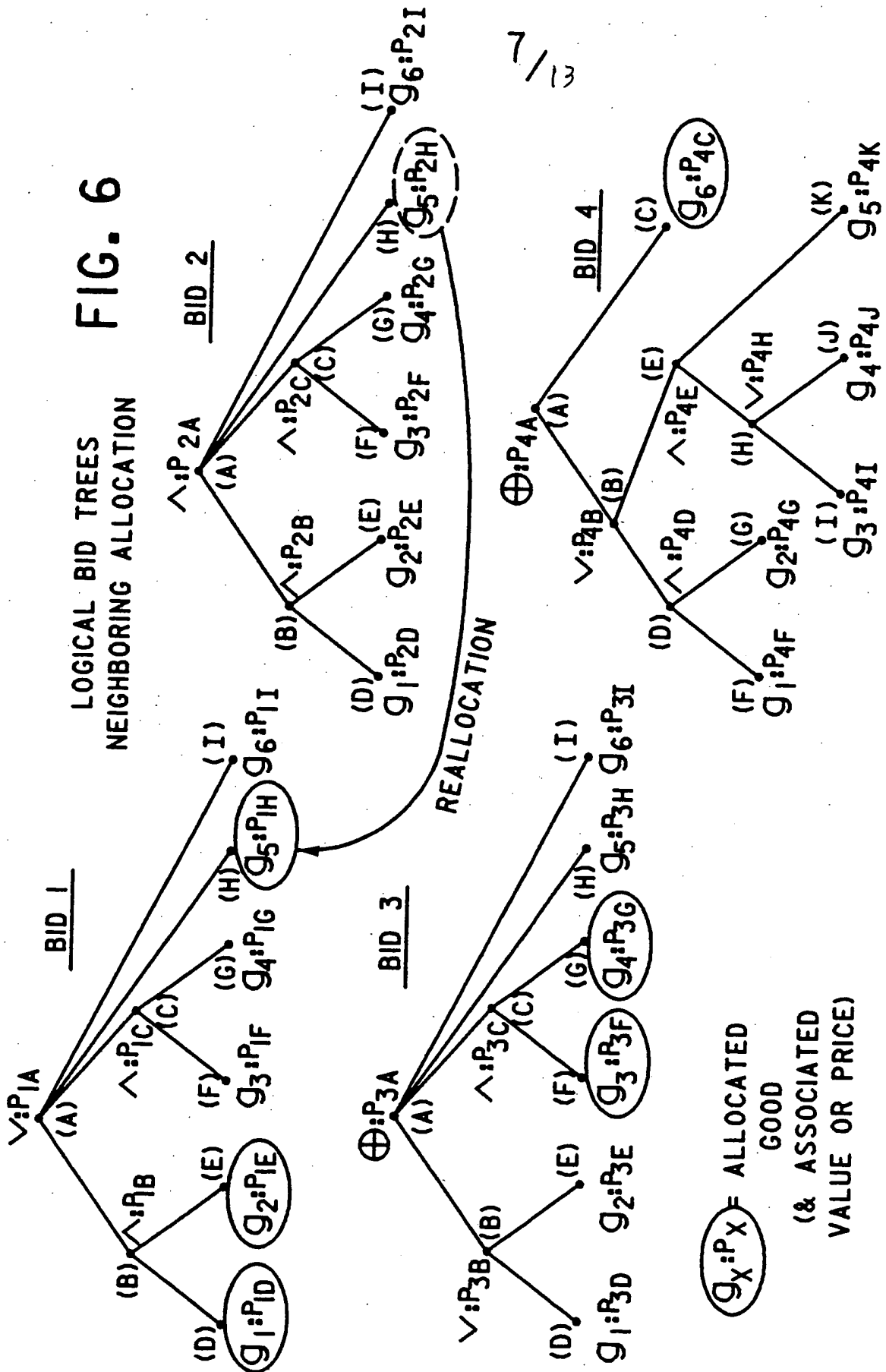
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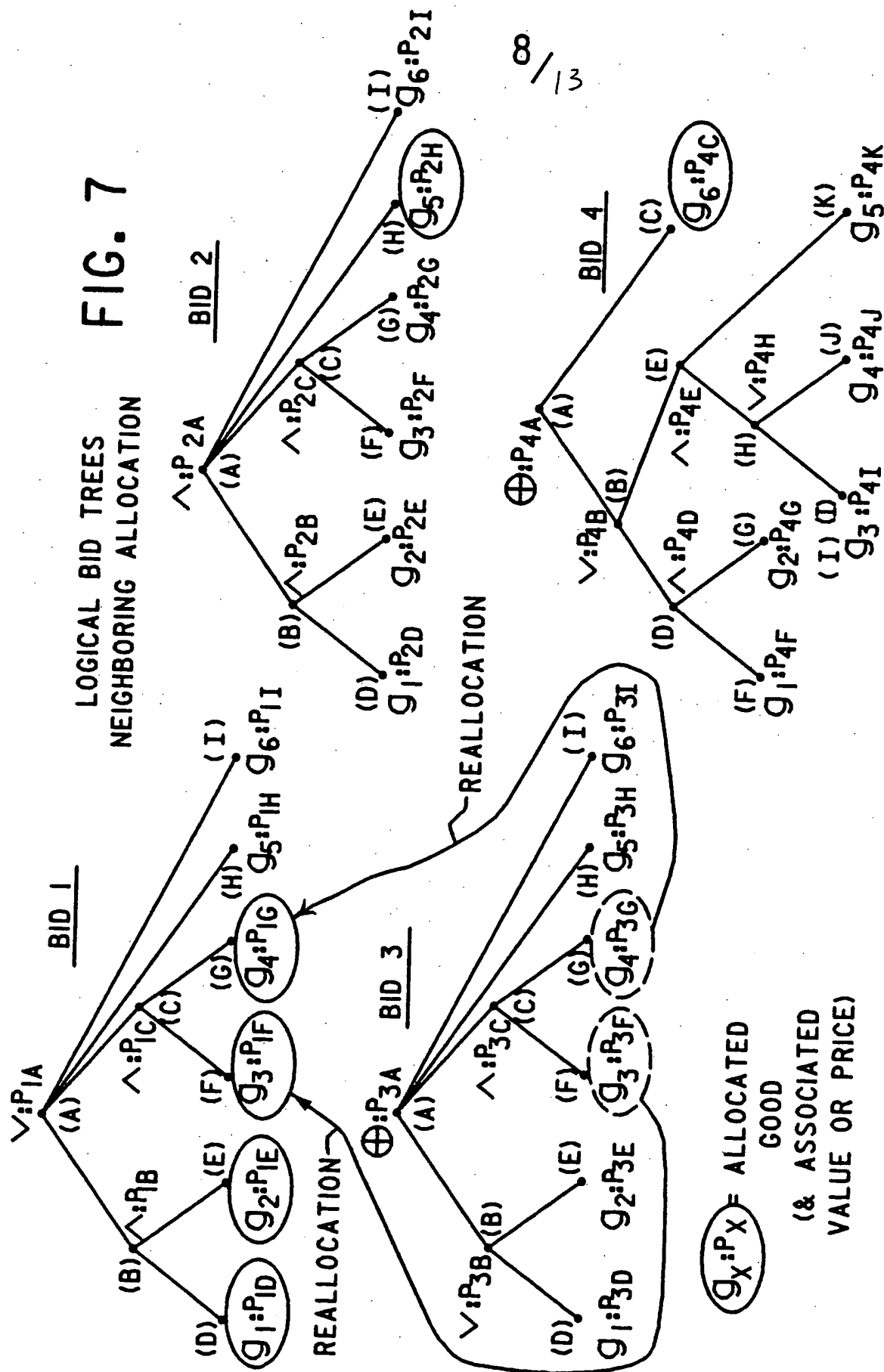
$$\begin{aligned}
 \text{BID 1} &= \left[ \left( \left( \left( G_1: P_{1D} \wedge G_2: P_{1E} \right) P_{1B} \right) \vee \left( G_3: P_{1F} \wedge G_4: P_{1G} \right) P_{1C} \right) \vee \left( G_5: P_{1H} \right) \vee \left( G_6: P_{1I} \right) \right] P_{1A} \\
 \text{BID 2} &= \left[ \left( \left( G_1: P_{2D} \wedge G_2: P_{2E} \right) P_{2B} \right) \wedge \left( G_3: P_{2F} \wedge G_4: P_{2G} \right) P_{2C} \right) \wedge \left( G_5: P_{2H} \right) \wedge \left( G_6: P_{2I} \right) \right] P_{2A} \\
 \text{BID 3} &= \left[ \left( \left( G_1: P_{3D} \vee G_2: P_{3E} \right) P_{3B} \right) \oplus \left( G_3: P_{3F} \wedge G_4: P_{3G} \right) P_{3C} \right) \oplus \left( G_5: P_{3H} \right) \oplus \left( G_6: P_{3I} \right) \right] P_{3A} \\
 \text{BID 4} &= \left[ \left( \left( \left( G_1: P_{4F} \wedge G_2: P_{4G} \right) P_{4D} \right) \vee \left( \left( G_3: P_{4I} \vee G_4: P_{4J} \right) P_{4E} \right) \wedge \left( G_5: P_{4K} \right) P_{4B} \right) \oplus \left( G_6: P_{4C} \right) \right] P_{4A}
 \end{aligned}$$

WHERE  $\wedge$  = AND,  $\vee$  = OR and  $\oplus$  = XOR;  
 $G$  = GOOD; and  
 $P$  = PRICE or VALUE.

FIG. 4







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Fig 8

$$\text{BID}_1 = \left[ \left( \underbrace{(G_1: P_{1D} \wedge G_2: P_{1E})}_{64} P_{1B} \right) \vee \left( \underbrace{(G_3: P_{1F} \wedge G_4: P_{1G})}_{66} P_{1C} \right) \vee \underbrace{(G_5: P_{1H})}_{72} \vee \underbrace{(G_6: P_{1I})}_{74} \right] P_{1A}$$

### Variables for Bid 1:

$$- X_{11}, X_{12}, X_{13}, X_{14}, X_{15} \text{ and } X_{16};$$
 where each  $x$  is Boolean, i.e.,  
 $\text{true}()$  if good is allocated,  
 otherwise  $\text{false}()$ .

—  $S_{60}, S_{62}, S_{64}, S_{66}, S_{68}, S_{70}, S_{72}, S_{74}$  and  $S_{76}$ ; where each  $S$  is Boolean, i.e., true(1) if corresponding is satisfied, otherwise false(0)

$V_{60}, V_{62}, V_{64}, V_{66}, V_{68}, V_{70}, V_{72}, V_{74}, V_{76}$ ; where each  $V$  is the value of the corresponding sub Bid  
 e.g.  $V_{60}$  = value of sub bid 60 (Integer or Real)

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Fig 9

$$\text{BID 3} = \left[ \underbrace{\left( \underbrace{\underbrace{(G_1, P_{3D})}_{80} \vee \underbrace{(G_2, P_{3E})}_{82} \right) P_{3B}}_{84} \oplus \underbrace{\left( \underbrace{(G_3, P_{3F})}_{86} \wedge \underbrace{(G_4, P_{3G})}_{88} \right) P_{3C}}_{90} \oplus \underbrace{(G_5, P_{3H})}_{92} \oplus \underbrace{(G_6, P_{3I})}_{94} \right] P_{3A}$$

Variables for Bid 3:

- $X_{31}, X_{32}, X_{33}, X_{34}, X_{35}$  and  $X_{36}$ ; where each  $X$  is Boolean, i.e., True (1) if good is allocated, otherwise False (0).  
Bid # Good #
- $S_{80}, S_{82}, S_{84}, S_{86}, S_{88}, S_{90}, S_{92}, S_{94}$  and  $S_{96}$ ; where each  $S$  is Boolean, i.e., True (1) if corresponding sub Bid is satisfied, otherwise False (0).  
Sub Bid Ref #
- $V_{80}, V_{82}, V_{84}, V_{86}, V_{88}, V_{90}, V_{92}, V_{94}$  and  $V_{96}$ ; where each  $V$  is the value of the corresponding sub Bid  
Sub Bid Ref #  
e.g.,  $V_{80}$  = value of sub Bid 80 (Real or Integer)
- $Z_{84}, Z_{90}, Z_{92}$  and  $Z_{94}$ ; where each  $Z$  is Boolean, i.e., True (1) if immediate sub Bid of Reference sub Bid contributes value thereto, otherwise False (0), e.g.,  $Z_{84}$  is True (1) if good 1 or 2 is allocated to sub Bid 80 or 82, respectively.

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Fig 10

$$Bids = \left[ k - \text{of} \left( \overbrace{(g_1 : p_{sb})}^{100}, \overbrace{(g_2 : p_{sc})}^{102}, \overbrace{(g_3 : p_{sd})}^{104} \right) p_{sa} \right]^{106}$$

where  $k$  is a real value  $\leq 2$ .

Variables for Bids:

- $x_{51}$ ,  $x_{52}$  and  $x_{53}$ ; where each  $x$  is Boolean true if good is allocated, otherwise Boolean false.  
 $\uparrow$  Bid#       $\uparrow$  Good#
- $s_{100}$ ,  $s_{102}$ ,  $s_{104}$  and  $s_{106}$ ; where each  $x$  is Boolean true if corresponding sub Bid satisfied, otherwise Boolean false.  
 $\underbrace{s_{100}}_{\substack{\text{sub Bid} \\ \text{Ref \#}}}$
- $v_{100}$ ,  $v_{102}$ ,  $v_{104}$  and  $v_{106}$ ; where each  $v$  is the value of the corresponding sub Bid.  
 $\underbrace{v_{100}}_{\substack{\text{sub Bid} \\ \text{Ref \#}}}$
- $n_{106}$ ; an integer or real value related to the number of satisfied sub bids of Bids

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FIG. 11(a)

(Atomic):

	<u>Equation 1</u>	<u>Equation 2</u>
Bid 1:	$S_{60} \leq X_{11}$ $S_{62} \leq X_{12}$ $S_{66} \leq X_{13}$ $S_{68} \leq X_{14}$ $S_{72} \leq X_{15}$ $S_{74} \leq X_{16}$	$V_{60} \leq P_{1D} * S_{60}$ $V_{62} \leq P_{1E} * S_{62}$ $V_{66} \leq P_{1F} * S_{66}$ $V_{68} \leq P_{1G} * S_{68}$ $V_{72} \leq P_{1H} * S_{72}$ $V_{74} \leq P_{1I} * S_{74}$
Bid 3:	$S_{80} \leq X_{31}$ $S_{82} \leq X_{32}$ $S_{86} \leq X_{33}$ $S_{88} \leq X_{34}$ $S_{92} \leq X_{35}$ $S_{94} \leq X_{36}$	$V_{80} \leq P_{3D} * S_{80}$ $V_{82} \leq P_{3E} * S_{82}$ $V_{86} \leq P_{3F} * S_{86}$ $V_{88} \leq P_{3G} * S_{88}$ $V_{92} \leq P_{3H} * S_{92}$ $V_{94} \leq P_{3I} * S_{94}$
Bid 5:	$S_{100} \leq X_{51}$ $S_{102} \leq X_{52}$ $S_{104} \leq X_{53}$	$V_{100} \leq P_{5B} * S_{100}$ $V_{102} \leq P_{5C} * S_{102}$ $V_{104} \leq P_{5D} * S_{104}$

FIG. 11(b)

(AND):

	<u>Equation 3</u>	<u>Equation 4</u>
Bid 1:	$2 * S_{64} \leq S_{60} + S_{62}$ $2 * S_{70} \leq S_{66} + S_{68}$	$V_{64} \leq P_{1B} * S_{64} + V_{60} + V_{62}$ $V_{64} \leq P_{1C} * S_{70} + V_{66} + V_{68}$
Bid 3:	$2 * S_{90} \leq S_{86} + S_{88}$	$V_{90} \leq P_{3C} * S_{90} + V_{86} + V_{88}$

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FIG. 11(c)

(OR and XOR):

	<u>Equation 5</u>	<u>Equation 6</u>
Bid 1:	$S_{76} \leq S_{64} + S_{70} + S_{72} + S_{74}$	$V_{76} \leq P_{1A} * S_{76} + V_{64} + V_{70} + V_{72} + V_{74}$
Bid 3:	$S_{84} \leq S_{80} + S_{82}$	$V_{84} \leq P_{3B} * S_{84} + V_{80} + V_{82}$
	$S_{96} \leq S_{84} + S_{90} + S_{92} + S_{94}$	$V_{96} \leq P_{3A} * S_{96} + V_{84} + V_{90} + V_{92} + V_{94}$

FIG. 11(d)

(XOR only):

	<u>Equation 7</u>	<u>Equation 8</u>
Bid 3:	$t_{84} + t_{90} + t_{92} + t_{94} \leq 1$	$V_{84} \leq \text{MAXVAL} * t_{84}$
		$V_{90} \leq \text{MAXVAL} * t_{90}$
		$V_{92} \leq \text{MAXVAL} * t_{92}$
		$V_{94} \leq \text{MAXVAL} * t_{94}$

FIG. 11(e)

(k-of) where k = 2:

Bid 5:	<u>Equation 9:</u>	<u>Equation 10:</u>
	$n_{106} \leq S_{100} + S_{102} + S_{104}$	$S_{106} * 2 \leq n_{106}$

Equation 11:

$$V_{106} \leq P_{5A} * S_{106} + V_{100} + V_{102} + V_{104}$$